Complete-simple distributive lattices] A construction of complete-simple distributive lattices   Computer Science Department
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Abstract

In this note we prove that there exist *complete-simple distributive lattices*, that is, complete distributive lattices in which there are only two complete congruences.

[

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1 Introduction

In this note we prove the following result:

[Sorry. Ignored `\begin{main} ... \end{main}`]

2 The construction

For the basic notation in lattice theory and universal algebra, see F. R. Richardson [] and G. A. Menuhin []. We start with some definitions:

Definition 1 *Let V be a complete lattice, and let p=[u,v] be an interval of V. Then p is called* complete-prime *if the following three conditions are satisfied:*

(1) u is meet-irreducible but u is not *completely meet-irreducible;*

(2) v is join-irreducible but v is not *completely join-irreducible;*

(3) [u,v] is a complete-simple lattice.

Now we prove the following result:

Lemma 1 *Let D be a complete distributive lattice satisfying conditions* (1) *and* (2)*. Then is a sublattice of ; hence is a lattice, and is a complete distributive lattice satisfying conditions* (1) *and* (2)*.*

[Sorry. Ignored `\begin{proof} ... \end{proof}`]

Corollary 1 *If D is complete-prime, then so is .*

The motivation for the following result comes from S.-K. Foo [].

Lemma 2 *Let τ be a complete congruence relation of such that*

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$$\langle 1,d \rangle \equiv \langle 1,1 \rangle \tau, \quad (1)$$

for some $d \in D$ with $d < 1$. Then $\tau = \iota$.

[Sorry. Ignored `\begin{proof} ... \end{proof}`]

3 The construction

The following construction is crucial to our proof of the Main Theorem:

Definition 2 *Let for $i \in I$ be complete distributive lattices satisfying condition* (2). *Their product is defined as follows:*

that is, is with a new unit element.

[Sorry. Ignored `\begin{notation} ... \end{notation}`]

See also E. T. Moynahan []. Next we verify:

Theorem 1 *Let for $i \in I$ be complete distributive lattices satisfying condition* (2). *Let τ be a complete congruence relation on . If there exist $i \in I$ and with such that for all ,*

(2)

then $\tau = \iota$.

[Sorry. Ignored `\begin{proof} ... \end{proof}`]

Theorem 2 *Let for $i \in I$ be complete distributive lattices satisfying conditions* (2) *and* (3). *Then also satisfies conditions* (2) *and* (3).

[Sorry. Ignored `\begin{proof} ... \end{proof}`]

The Main Theorem follows easily from Theorems and .

References

[] Soo-Key Foo, *Lattice Constructions*, Ph.D. thesis, University of Winnebago, Winnebago, MN, December 1990.

[] George A. Menuhin, *Universal Algebra*, D. van Nostrand, Princeton-Toronto-London-Melbourne, 1968.

[] Ernest T. Moynahan, *On a problem of M. H. Stone*, Acta Math. Acad.Sci. Hungar. **8** (1957), 455–460.

[] , *Ideals and congruence relations in lattices. II*, Magyar Tud. Akad. Mat. Fiz. Oszt. Közl. **9** (1957), 417–434 (Hungarian).

[] Ferenc R. Richardson, *General Lattice Theory*, Mir, Moscow, expanded and revised ed., 1982 (Russian).